MINISTRY OF EDUCATION AND SCIENCE OF RUSSIA
Federal State Budgetary Educational Institution of Higher Education
Chelyabinsk State University (FSBEI HE CSU)

APPROVING
Vice-Rector for Academic Affairs
_____ A.A. Salamatov
"22" 02 _____, _____ 2026

ENTRANCE EXAMINATION PROGRAM FOR ADMISSION TO THE MASTER'S DEGREE PROGRAM

Field of study
01.04.01 Mathematics

Master's Program
Fractional Differential Equations

for applicants entering educational program of higher education – master's program

Chelyabinsk 2026

The program was compiled in accordance with the requirements of the Federal State Educational Standard for Higher Education – Master's degree in 01.04.01 Mathematics (approved by Order No. 12 of the Ministry of Education and Science of the Russian Federation dated 01/10/2018).

The entrance test program was approved and recommended by the Academic Council of the Faculty of Mathematics
Protocol dated February 19, 2026 No. 8

Acting Dean of the
Faculty of Mathematics _____ A.S. Skorynin

Head of the Department
of Mathematical Analysis _____ V.E. Fedorov

Chairman of the Subject Committee,
Associate Professor of the Department
of Mathematical Analysis _____ A.V. Nagumanova

CONTENT

1 GENERAL PROVISIONS

This program is based on the federal state educational standards of higher education for bachelor's and specialist's degree programs, and defines the general content of the exam for admission to higher education programs – master's degree program in the field of study of 01.04.01 Mathematics at the Federal State Budgetary Educational Institution of Higher Education "Chelyabinsk State University" (hereinafter referred to as the University).

The comprehensive exam is aimed at assessing the knowledge of applicants obtained during the development of bachelor's and (or) specialist's degree programs, and at selecting among applicants the most capable and prepared to master master's degree programs in the fields of 01.04.01 Mathematics.

The comprehensive examination in the specialty is conducted as a single entrance test within the framework of several competitions (according to the appropriate forms and fundamentals of education) within one group of master's degree programs and is taken once.

The examination tasks for the comprehensive exam in the program include three practical tasks in the following sections: algebra, geometry, mathematical analysis, differential equations, probability theory, numerical methods.

The entrance test is conducted in English.

The entrance test is conducted in person or using remote technologies in cases stipulated by the Admission Rules.

Вступительное испытание проводится очно или с использованием дистанционных технологий в случаях, предусмотренных Правилами приема.

2 CONTENTS OF SECTIONS

Section 1 Algebra

1) MATRICES AND DETERMINANTS. Definitions of the determinant and its basic properties. The theorem on the decomposition of the determinant by the elements of a row (column). The criterion of reversibility of the matrix.

2) ALGEBRA OF POLYNOMIALS. The largest common divisor of two polynomials (Euclid's algorithm).
3) LINEAR (VECTOR) SPACES AND SYSTEMS OF LINEAR EQUATIONS. Linear dependence and independence of vector systems. Subspaces. The linear shell of a vector system. Basis and dimension. The theorem on the dimension of the sum of two subspaces. The rank theorem of a matrix. The theorem on the dimension of the solution space of a homogeneous system of linear equations.
4) LINEAR TRANSFORMATIONS. The kernel and the image of the linear mapping. The theorem on the dimensions of the kernel and the image. The matrix of linear transformation of a finite-dimensional vector space. Eigenvalues and eigenvectors of a linear transformation, a theorem on the relation of the eigenvalues of a linear transformation to the roots of its characteristic polynomial.
5) EUCLIDEAN AND UNITARY SPACES. The orthogonalization theorem. The orthonormal basis.

Examples of task types

1) Calculation of the determinant. Actions with matrices. Calculating the inverse matrix. Kramer's formula. The Gauss method for solving linear algebraic systems.
2) A division algorithm with a remainder in a ring of polynomials with one unknown. Euclid's algorithm.
3) Methods for calculating the rank of a matrix. The fundamental system of solutions for homogeneous system of equations.
4) Finding the eigenvalues and eigenvectors of a linear transformation.
5) The process of orthogonalization of the vector system of Euclidean space. Calculating the orthogonal projection.

Section 2 Geometry

1) VECTORS. Collinearity and coplanarity of vectors. The coordinates of the vector in the affine coordinate system. Scalar and vector products. The properties, the geometric meaning of these products, and their expression in coordinates.
2) A STRAIGHT LINE AND A PLANE. A theorem on the general equation of a plane in space. The normal vector and the theorem on the distance from a point to a plane.
3) SECOND-ORDER CURVES. Definition and derivation of canonical equations of ellipse, hyperbola and parabola. Determining the type of curve by a second-order equation on a plane.

Examples of task types

1) Dividing a segment in a given ratio. The distance between two points. The volume of the parallelepiped. Calculation of scalar, vector, and mixed products based on multiplier coordinates.
2) The main types of equations of a straight line and a plane. The angle between two straight lines. The distance from the point to the plane and to the straight line. The relative position of the planes
3) Reduction of the equation of the second-order curve to the canonical form. Canonical equations of surfaces of the second order.

Section 3 Mathematical Analysis

1) THEORY OF THE LIMIT. Sequence limit and function limit. The theorem on the existence of an exact upper bound.

2) CONTINUOUS FUNCTIONS. The Bolzano–Cauchy theorem on the intermediate value of a function. The Weierstrass theorem on the largest and smallest value of a function.
3) DIFFERENTIABLE FUNCTIONS. Rolle and Lagrange theorems. The Taylor formula with a residual term in Lagrange form.
4) INTEGRATION. The Riemann integral. The integrability theorem of a continuous function. The theorem on the continuity and differentiability of an integral with a variable upper limit. The Newton–Leibniz formula.
5) FUNCTIONS OF MANY VARIABLES. Differentiability of functions of many variables. The theorem on sufficient conditions for differentiability of a function.
6) FUNCTIONAL SEQUENCES AND SERIES. Uniform and pointwise convergence of functional sequences and series. Targeted differentiation and integration of functional series. Power series. The Cauchy–Hadamard theorem on the radius of convergence of a power series. Targeted differentiation and integration of power series.

Examples of task types

1) Properties of function limits. Remarkable limits. Calculation of function limits using Lopital's rule, Taylor's formula.
2) The study of functions using derivatives. Extremum, convexity. Integration methods: integration by parts, substitution of variables, the Newton–Leibniz formula. Calculation of improper integrals.
3) Calculation of partial derivatives and differentials of complex functions and functions specified implicitly.
4) Investigation of the convergence of numerical series (signs of comparison, Cauchy, Dalembert, Dirichlet, Weierstrass). Decomposition of functions into power series. Calculation of the radius of convergence of a power series.
1). 2). 3). 4).

Section 4 Differential Equations

1) Linear differential equation of the n-th order with constant coefficients.
2) The method of constant variation for finding a solution to an inhomogeneous linear differential equation of the first order.

Examples of task types

1) Equations with separable variables. Homogeneous equations. Linear equations. Bernoulli's equations. Equations in full differentials.
2) Linear differential equations with constant coefficients (inhomogeneous with a special right-hand side).

Section 5 Probability Theory

1) Random events. The probability of the event. A discrete probability space. Continuous probability space.
2) Conditional probability. The formula of total probability and the Bayesian formula.
3) Bernoulli test scheme. The Bernoulli formula. The most unlikely number of successes in Bernoulli trials.
4) Random variables. The distribution function. Probability density. Mathematical expectation and variance, initial and central moments, mode, median.
5) The basic laws of the distribution of discrete random variables – binomial, Poisson, geometric.

6) The normal distribution law. Mathematical expectation and variance of a normal random variable. Properties of a random variable with a normal distribution law.
7) The central limit theorem and its significance.

Examples of task types

1) Calculate the probability of an event using combinatorial reasoning.
2) Calculate the conditional probability of the event.
3) Find the mathematical expectation and variance of a random variable.

Section 6 Numerical Methods

1) Estimation of calculation errors through the error of the source data, significant figures.
2) Newton's method of numerical solution of nonlinear equations.
3) Euler's method and Runge–Kutta methods for numerical solution of differential equations.
4) The run method for solving linear systems of equations, iterative methods for solving linear and nonlinear equations.
5) Interpolation by splines and interpolation polynomials in the form of Lagrange and Newton.

Examples of task types

1) Calculate the first few iterations of Newton's method for a given equation.
2) Estimate the number of steps of the iterative solving method for linear systems of equation to achieve the specified accuracy.
3) Construct an interpolation polynomial for a given tabular function.

3 THE FORM OF ENTRANCE EXAMINATIONS AND THE TYPES OF TEST (OR CREATIVE) TASKS

Entrance examinations are conducted in a combined form: a written exam and an interview. The written exam is conducted on tickets, each of which contains three practical questions in the form of tasks in the following sections: algebra, geometry, mathematical analysis, differential equations, computer science and programming languages, data structures and algorithms, probability theory, numerical methods, discrete mathematics. The subject committee checks the answer and conducts an interview with the student, asking additional questions both about the program and about previous scientific research conducted by the applicant.

4 THE RULES OF THE ENTRANCE TEST

The applicant is given an answer form, an entrance test exam ticket. The applicant responds to the ticket in writing within 60 minutes. The subject committee checks the answer and conducts an interview with the student, asking additional questions both about the program and about previous scientific research conducted by the applicant. During the test, it is forbidden to get up, move around, talk, exchange anything, use reference materials, mobile phones and other means of communication, photo and video equipment, calculators, tablets and personal computers. It is allowed to use pens, pencils, and rulers. Working forms are submitted to a member of the subject committee upon completion of the test.

5 CRITERIA FOR EVALUATING THE PERFORMANCE OF THE EXAM ASSIGNMENT

General requirements for completing tasks

Each of the three tasks requires a written answer to the ticket's questions. The text of the response should be neatly designed, logically stated, contain correct calculations.

Assessment scale

The maximum number of points for the entrance test is 100 points. The minimum number of points for successful completion of the entrance test, regardless of the admission conditions, corresponds to the minimum points approved by the Rules for the current year.

Criteria for evaluating the results of the entrance test

80-100 points – the applicant has discovered a comprehensive, systematic and in-depth knowledge of the educational and program material, exhaustively, consistently, competently and logically presents it, does not find it difficult to answer when modifying the assignment, freely copes with the tasks set, shows knowledge of the monographic material, correctly justifies the decisions made, possesses versatile skills and techniques for performing practical work, discovers the ability to independently summarize and present the material without making mistakes, He understood the relationship between the basic concepts of discipline and their importance for acquiring a profession.

60-79 points – the applicant firmly knows the educational and program material, presents it competently and to the point, does not allow significant inaccuracies in answering the question, can correctly apply theoretical provisions and possesses the necessary skills when performing practical tasks.

40-59 points – the applicant has mastered only the basic material, but does not know individual details, makes inaccuracies, insufficiently correct formulations, violates the sequence in the presentation of the program material and has difficulty in performing practical tasks.

0-39 points – the applicant does not know a significant part of the program material, makes significant mistakes, and performs practical work with great difficulty.

6 LIST OF RECOMMENDED SOURCES FOR PREPARATION FOR ENTRANCE TEST

Primary:

1. Aleksandrov, P. S. Course of Analytical Geometry and Linear Algebra: a textbook for universities / P. S. Aleksandrov. - St. Petersburg: Lan, 2025. - 512 p. - URL: https://e.lanbook.com/book/448691

2. Bermant, A. F. A Short Course in Mathematical Analysis: a textbook for universities / A. F. Bermant, I. G. Aramanovich. - St. Petersburg: Lan, 2022. - 736 p. - URL: https://e.lanbook.com/book/210707

Additional:

1. Gmurman, V. E. Probability Theory and Mathematical Statistics: a textbook for secondary vocational education / V. E. Gmurman. - Moscow: Yurait, 2024. - 479 p. - URL: https://urait.ru/book/teoriya-veroyatnostey-i-matematicheskaya-statistika-536720
2. Demidovich, B. P. Collection of Problems and Exercises in Mathematical Analysis: a textbook for universities / B. P. Demidovich. - St. Petersburg: Lan, 2024. - 624 p. - URL: https://e.lanbook.com/book/426251
3. Emelianov, G. V. Problem Book on Probability Theory and Mathematical Statistics: a textbook for universities / G. V. Emelianov, V. P. Skitovich. — Saint Petersburg: Lan, 2022. — 332 p. — URL: https://e.lanbook.com/book/206273
4. Collection of Problems in Algebra: in 2 volumes / edited by A. I. Kostrikin. - Moscow: Fizmatlit, 2007. - Vol. 1. - Part I and II. Fundamentals of algebra. Linear algebra and geometry. - 263 p. - URL: https://biblioclub.ru/index.php?page=book&id=82941
5. Ter-Krikorov, A. M. Course in Mathematical Analysis: a textbook for universities / A. M. Ter-Krikorov, M. I. Shabunin. - Moscow: Fizmatlit, 2001. - 668 p. - URL: https://biblioclub.ru/index.php?page=book&id=83198. – ISBN 5-9221-0008-4
6. Fichtenholz, G. M. Course of Differential and Integral Calculus: textbook for universities: in 3 volumes / G. M. Fichtenholz. - St. Petersburg: Lan. - Volume 1 - 2024. - 608 p. - URL: https://e.lanbook.com/book/403391
7. Kostrikin, A. I. Introduction to Algebra: a textbook for universities / A. I. Kostrikin. - Moscow : MCNO, 2009. - Part 1. Fundamentals of Algebra. - 273 p. - URL: https://biblioclub.ru/index.php?page=book&id=63140
8. Trench, W. F. Elementary Differential Equations: Books and Monographs / W. F. Trench. - 2013. Book 8. - URL: http://digitalcommons.trinity.edu/mono/8
9. Trench, W. F. Introduction to Real Analysis: Books and Monographs / W. F. Trench. - 2013. Book 7. - URL: http://digitalcommons.trinity.edu/mono7
10. Jezek, J. Universal Algebra / J. Jezek - 2008.
11. Geometry. - University of South Carolina.
12. Kaisare, N.S. Computational Techniques for Process Simulation and Analysis Using MATLAB / N. S. Kaisare.

List of resources from the information and telecommunication network "Internet" recommended for preparation for entrance exams:

1) Internet University of Information Technology http://www.intuit.ru/
2) eLIBRARY.RU http://elibrary.ru
3) Math-Net.Ru http://www.mathnet.ru/

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